Title: Error Correcting Codes

Brief Overview:

This lesson is an introduction to error correcting codes. It discusses the need for error correction and a few examples of error correcting codes. Students will then work on several types of error correction and discuss the advantages and limitations of some of the systems. Systems include binary 1-error detecting and correcting codes and the bar code used to represent zip codes on magazine subscription cards.

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will work to understand how errors can occur in data transmission and the need for code to correct these errors. They will then work through examples to see how some codes have been developed to solve the problem of errors.

- **Mathematics as Communication**
  Students will understand how codes are used in data transmission and the relationship between math and error correcting codes. Students should then be able to communicate why there is a need for error correcting codes, and be able to explain some examples.

- **Mathematics as Reasoning**
  Students will discuss bit flipping in data transmission and use error correcting codes to reason about the concept of minimum distance to code words, leading to discussion of the hypercube and its use in creating error correcting codes.

- **Mathematical Connections**
  Students will examine the relationship of math and geometry to cryptography and error correcting codes.

- **Number and Number Relationships**
  Students will understand addition in different moduli (2 and 10).

- **Geometry**
  Students will investigate the relationship between the hypercube and vector norms, and the relationship of these concepts to error correcting codes.

Grade/Level:

Grades 6-8

Duration/Length:

1-2 class periods

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Binary number system
- Use of computers and the binary number system in coding

Objectives:

Students will:

- understand how error correcting codes work.
- be able to use a key to correct code based on the concept of minimum distance.
- understand the difference between error detection and error correction.

Materials/Resources/Printed Materials:

- Attached handouts and notes from the instructor
- Subscription card (attached)

Development/Procedures:

The teacher should first begin a discussion of how errors can occur during data transmission, leading to the need for error correcting codes. They should discuss the need to both identify and correct errors. The teacher should discuss some common errors that humans can easily correct, such as spelling mistakes in text. Students should then try to correct the paragraph on the first worksheet and answer the two associated questions. The teacher should then lead the discussion to the use of binary in data transmission. Examples of error detection can be presented, e.g., having 5-bit code words with a 6th bit as a parity bit. Students should discuss this concept and realize that this will only detect one error, but not two. This leads to a discussion of how codes can be used to correct themselves, using the concept of distance in terms of vector norms. The students should then work on the second handout relating to error correction in binary codes. This leads to a discussion of distance in relation to the hypercube on the third handout. Students should discuss advantages (self correcting) and disadvantages (requires longer codewords for error correction) of such systems. Finally, students can examine the bars at the bottom of a magazine subscription card (used to represent zip codes) to decode the system used to represent the ten digits as well as using the last codeword as an error detection device (all ten digits should add to 0 modulo 10).

Performance Assessment:

Students will be assessed based on class discussion and on their performance on the handouts.

Extension/Follow Up:

Some extensions would include further discussion of distance as it related to error correction and discussing how two errors could be corrected. Students should also investigate properties of UPCs.

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CORRECTING ERRORS

- Errors occur when data is sent.
  - Bad transmission medium (e.g., bad weather)
  - Data sent at too high a speed for a medium to handle

- Many times, it is necessary to determine if there are errors (and/or correct them).

- Some errors are obvious-- as in garbled paragraph worksheet (Handout 1).

- Not all errors are obvious-- example, binary data.

- Parity bits are used to detect errors.
  - Example: 5-bit code words--add a 6th bit--sum of 1st five
  - This will DETECT errors; not correct them (sometimes can do next one by eyeball).

- Codewords can be created with “distances”between them.
  - “Distance” is the number of differences between codewords.
  - If error occurs, pick codeword “closet in distance” to garbled codeword.

    Examples: 11111 00000 -- codewords
    Garbled data: 11110 10111 00100 -- can be corrected
    2nd set of data: 10101 01100 -- corrects to 11111 00000

- DISADVANTAGE: Requires more data to include parity bit or longer codewords.

- Handout 5-bit codeword graph and worksheets. (Handout 2)

CORRECTING ERRORS

1. How many errors can you find in the below paragraph?
2. Can you correct enough errors to answer the question asked?

Old McDonald decidez one day to count akl of his farm animals. He fouxd twenty-three caws. Next, he saw eughteen shiep. He went pnto his yarn and counted hixty-five horsts. Out inn the sud he found twllve pigs. Ge went arond hss mouse and taw twirty chhckens. Fanally, ke hounted geven docks. So sow kany anisals ddd ha haae ih axll?

CORRECTING ERRORS
(Answer Sheet)

1. How many errors can you find in the below paragraph?
2. Can you correct enough errors to answer the question asked?

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23 cows
18 sheep
65 horses
12 pigs
30 chickens
7 ducks
155 animals in all

CORRECTING ERRORS

You receive a message that uses the following coding:

10001 - T
01101 - E
10110 - S
01010 - A

You have been told to expect some errors in the message. Decode the message, and correct any errors you find.

MESSAGE: (ONE WORD ON EACH LINE)

11110 10001 00010 10001 00101

10001 01111 01010

10011 01010 10100 10001 11101

10001 01100 10110 10101

01010 11001

10110 00101 01010 10000

01010

CORRECTING ERRORS
(Answer Sheet)

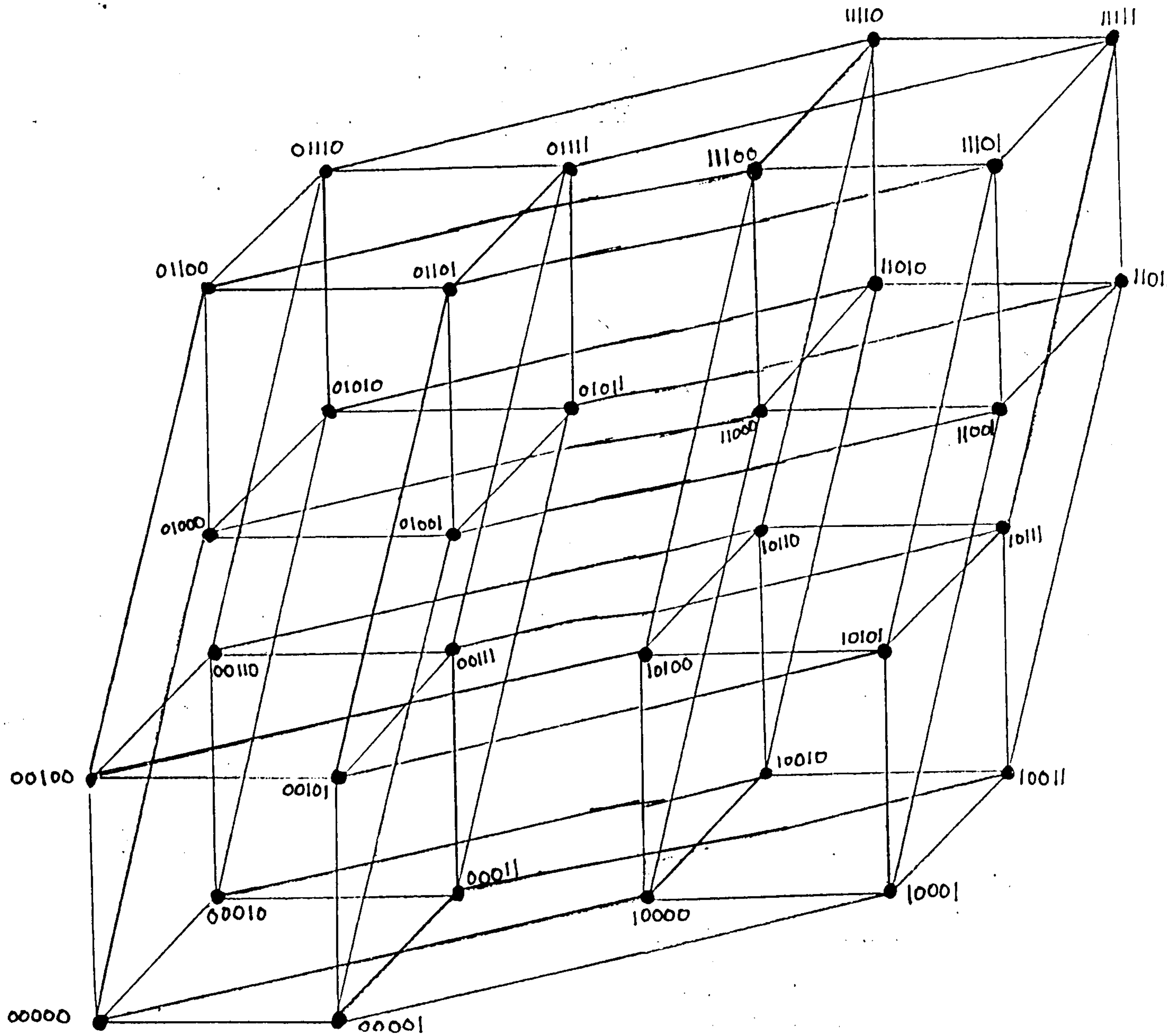
You receive a message that uses the following coding:

10001 - T
01101 - E
10110 - S
01010 - A

You have been told to expect some errors in the message. Decode the message, and correct any errors you find.

MESSAGE: (ONE WORD ON EACH LINE)

S	T	A	T	E
$1\underline{1}110$	10001	$0\underline{0}010$	10001	$0\underline{0}101$
T	E	A		
10001	$011\underline{1}1$	01010		
T	A	S	T	E
$100\underline{1}1$	01010	$101\underline{0}0$	10001	$\underline{1}1101$
T	E	S	T	
10001	$0110\underline{0}$	10110	$10\underline{1}01$	
A	T			
01010	$1\underline{1}001$			
S	E	A	T	
10110	$0\underline{0}101$	01010	$1000\underline{0}$	
A				
01010				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00010
10000
10001
00000
00001

ERROR DETECTION - BAR CODES

EXAMPLE:

On magazine subscription cards, barcodes represent zip code digits as follows:

0

1

2

3

4

5

6

7

8

9

When machine reads barcode, it:

(1) Removes first and last bars (both should be long)
(2) Separates remaining bars into groups of 5
(3) The first 9 groups translate to the zip code digits
(4) The last group is a “parity”/check group:
Add the 9 digits from zip codes (MOD 10)
Add the above sum to the check group
The new sum should be 0 (MOD 10)
* Instead of “MOD”, students can be told to consider only the last digit

Let students try sample subscription card or other cards that may be provided.

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